

# PRIMORDIAL FRACTAL DENSITY PERTURBATIONS AND STRUCTURE FORMATION IN THE UNIVERSE : 1-DIMENSIONAL COLLISIONLESS SHEET MODEL

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Two-point correlation function of galaxy distribution shows that the structure in the present Universe is scale-free up to a certain scale (at least several tens Mpc), which suggests that a fractal structure may exist. If small primordial density fluctuations have a fractal structure, the present fractal-like nonlinear structure below the horizon scale could be naturally explained. We analyze the time evolution of fractal density perturbations in Einstein-de Sitter universe, and study how the perturbation evolves and what kind of nonlinear structure will come out. We assume a one-dimensional collisionless sheet model with initial Cantor-type fractal perturbations. The nonlinear structure seems to approach some attractor with a unique fractal dimension, which is independent of the fractal dimensions of initial perturbations. A discrete self-similarity in the phase space is also found when the universal nonlinear fractal structure is reached.

## 1 Introduction

The present Universe shows a variety of structures. The galaxies are not distributed randomly in the Universe. Totsuji and Kihara <sup>1</sup> and Peebles <sup>2</sup> showed that the observed two-point correlation function  $\xi(r)$  is given by a power law with respect to a distance  $r$  as  $\xi(r) \sim r^{-\gamma}$  with  $\gamma \sim 1.8$ . The recent galaxy surveys also agree with this result, i.e. the power  $\gamma$  is nearly equal to 1.8. This may imply that the present distribution of galaxies is fractal.

However, the observation of Cosmic Microwave Background Radiation (CMBR) has revealed that the Universe in the recombination era is homogeneous and isotropic at least in very large scale. Although CMBR observation seems to be more reliable, we should not decide yet whether the large scale structure of the Universe is really fractal up to the horizon scale or not.

Since it seems true that the galaxy distribution is really fractal up to a certain scale, one may ask how such a structure is formed in the evolution of the Universe. One of the most plausible explanations is that the nonlinear dynamics of the perturbations will provide such a scale-free structure during the evolution of the Universe. The pioneering work to explain the power-law behavior in nonlinear stage has been done by Davis and Peebles <sup>3</sup>. They assume a self-similar evolution of density fluctuation and some additional condition, i.e. a physical velocity  $\dot{r}$  vanishes in nonlinear regime. Then they showed a relation between the power index  $\gamma$  of two-point correlation function and that of initial power spectrum  $n$  as  $\gamma = 3(n+3)/(n+5)$ . If we have  $n = 0$ , then we find that  $\gamma = 1.8$ . Since we do not know the stability of those solutions, in order to find which value of  $\gamma$  is most

likely, we should study the dynamics of density fluctuations in other methods.

As for a fractal structure in the Universe, one may ask another question. Did the Universe not have any non-trivial structure such as a fractal in the initial density fluctuations? In the conventional approaches, initial density perturbations are usually assumed to be given by a power-law (or a power-law-like) spectrum with random Gaussian phase. Although such initial conditions may provide the presently observed nonlinear scale-free structure via nonlinear dynamics, no one has shown whether such a structure is fractal or not, and if yes, what kind of fractal structure comes out. To provide a fractal structure in the present Universe, we may adopt an alternative scenario, in which primordial density fluctuations have already a fractal-like structure in the beginning. The properties of an initial fractal may be preserved during the evolution of the Universe, then nonlinear fractal structure will be formed. In this scenario, several natural questions may arise. How does such a primordial fractal perturbation evolve into nonlinear regime? Will any properties of the initial fractal be preserved during the evolution of the Universe, or not? If not, what kind of nonlinear structure will come out at present? Is there any fundamental difference in the structure formation process between a conventional density perturbation and the present fractal one? In order to answer those questions, we study the time evolution of the initial density fluctuations with a fractal structure in Einstein de-Sitter universe.

Since we are interested in a fractal structure, a quite high resolution is required in our calculation. As we discussed,  $N$ -body simulation may not have enough resolution in the present state of computer development, unless we develop some skillful method. So, in this paper, we consider only a very simple toy model, which is a one-dimensional (1-D) sheet model, in order to get some insight into the questions raised in the above. To set up primordial fractal density perturbations, we distribute  $N$  sheets initially by some systematic rule, i.e. we apply a Cantor set or random Cantor-type set. Mathematically, in order to construct a Cantor set, the procedure must be repeated an infinite number of times, but it is not practically possible to set up such initial data. We therefore stop the procedure at a certain point, i.e. the initial set is given by several times removing line segments with a given ratio <sup>4</sup>. This could be justified because an infinite scale-free structure never exists in the real Universe. Since we study a 1-D sheet-model, the motion of each sheet is described by an analytic solution <sup>5</sup>, which guarantees enough resolution to analyze a fractal structure.

In this situation, we find a kind of attractor with a universal fractal dimension ( $\sim 0.9$ ) as the fluctuations evolve into nonlinear regime <sup>6</sup>.

## References

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